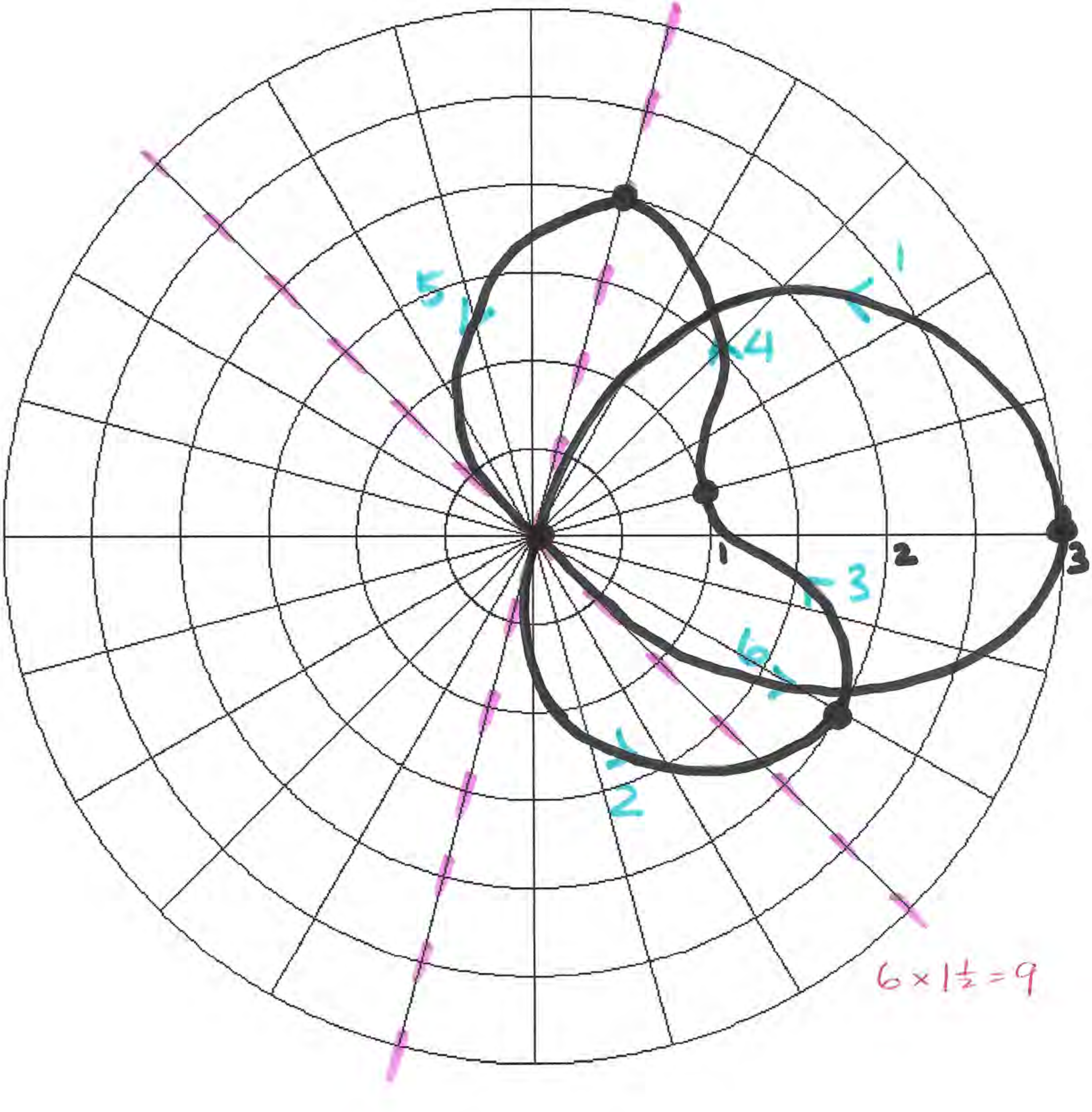
[2] MAX/MIN/ZERO AT $\theta = 0, \frac{5\pi}{12}, \frac{10\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}, 2\pi$

2	$\theta = 0 \to \frac{5\pi}{12}$	$\lvert r \rvert$ DECR $\to$ SPIRAL IN TO POLE	
2	$\frac{5\pi}{12} \to \frac{10\pi}{12}$	INCR	OUT TO 2
2	$\frac{10\pi}{12} \to \frac{13\pi}{12}$	DECR	IN TO 1
2	$\frac{13\pi}{12} \to \frac{17\pi}{12}$	INCR	OUT TO 2
2	$\frac{17\pi}{12} \to \frac{21\pi}{12}$	DECR	IN TO POLE
2	$\frac{21\pi}{12} \to 2\pi$	INCR	OUT TO 3



1
2
3
1
2
3
4
5
6
6 × 1½ = 9

[3][a] $r = \sqrt{(0 - -3)^2 + (0 - -4)^2} = 5$

1 $(x+3)^2 + (y+4)^2 = 25$

2 $(r\cos\theta + 3)^2 + (r\sin\theta + 4)^2 = 25$

$r^2\cos^2\theta + 6r\cos\theta + 9 + r^2\sin^2\theta + 8r\sin\theta + 16 = 25$

3 $r^2 + 6r\cos\theta + 8r\sin\theta = 0$

$r(r + 6\cos\theta + 8\sin\theta) = 0$

$r = 0$ OR $r = -6\cos\theta - 8\sin\theta$ 2

[b] $r = 0 = -6\cos\theta - 8\sin\theta$ 2

$8\sin\theta = -6\cos\theta$

2 $\tan\theta = -\frac{3}{4}$ $\longrightarrow$ $\theta = \pi - \tan^{-1}\frac{3}{4}$ OR $2\pi - \tan^{-1}\frac{3}{4}$ 2

[c] 3 $\displaystyle\int_{\frac{\pi}{2}}^{\pi-\tan^{-1}\frac{3}{4}} \sqrt{(-6\cos\theta-8\sin\theta)^2+(6\sin\theta-8\cos\theta)^2}\,d\theta$ 3

$$= \int_{\frac{\pi}{2}}^{\pi-\tan^{-1}\frac{3}{4}} \sqrt{36\cos^2\theta+96\cos\theta\sin\theta+64\sin^2\theta+36\sin^2\theta-96\sin\theta\cos\theta+64\cos^2\theta}\,d\theta$$

$$= \int_{\frac{\pi}{2}}^{\pi-\tan^{-1}\frac{3}{4}} \overset{3}{\sqrt{36+64}}\,d\theta = 10\left(\pi-\tan^{-1}\frac{3}{4}-\frac{\pi}{2}\right)$$

$$= 5\pi-10\tan^{-1}\frac{3}{4}$$ 2

[d] $\frac{1}{2}\int_{\frac{\pi}{2}}^{\pi-\tan^{-1}\frac{3}{4}} 2\left(36\cos^2\theta + 96\cos\theta\sin\theta + 64\sin^2\theta\right)d\theta$

$= \int_{\frac{\pi}{2}}^{\pi-\tan^{-1}\frac{3}{4}} \left(18\left(\frac{1}{2}(1+\cos 2\theta)\right) + 24\sin 2\theta + 32\left(\frac{1}{2}(1-\cos 2\theta)\right)\right)d\theta$

$= \int_{\frac{\pi}{2}}^{\pi-\tan^{-1}\frac{3}{4}} \left(25 - 7\cos 2\theta + 24\sin 2\theta\right)d\theta$

$= \left(25\theta - \frac{7}{2}\sin 2\theta - 12\cos 2\theta\right)\Big|_{\frac{\pi}{2}}^{\pi-\tan^{-1}\frac{3}{4}}$

$= 25\left(\pi - \tan^{-1}\frac{3}{4} - \frac{\pi}{2}\right) - \frac{7}{2}\left(\sin\left(2\pi - 2\tan^{-1}\frac{3}{4}\right) - 0\right)$
$\quad - 12\left(\cos\left(2\pi - 2\tan^{-1}\frac{3}{4}\right) - -1\right)$

$= \frac{25\pi}{2} - 25\tan^{-1}\frac{3}{4} + \frac{7}{2}\sin\left(2\tan^{-1}\frac{3}{4}\right) - 12\cos\left(2\tan^{-1}\frac{3}{4}\right) - 12$

$= \frac{25\pi}{2} - 25\tan^{-1}\frac{3}{4} + \frac{7}{2}\cdot\frac{24}{25} - 12\cdot\frac{7}{25} - 12$

$= \frac{25\pi}{2} - 25\tan^{-1}\frac{3}{4} - 12$

$\sin\left(2\tan^{-1}\frac{3}{4}\right) = \sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$

$\alpha = \tan^{-1}\frac{3}{4} \rightarrow \tan\alpha = \frac{3}{4},\ \alpha \in Q_1$

$\cos\left(2\tan^{-1}\frac{3}{4}\right) = \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$

[4] $r = \cos^2\theta - \sin^2\theta + \cos\theta$ 3

$r = \frac{x^2}{r^2} - \frac{y^2}{r^2} + \frac{x}{r}$ 4

$r^3 = x^2 - y^2 + xr$

4 $(x^2+y^2)\sqrt{x^2+y^2} = x^2 - y^2 + x\sqrt{x^2+y^2}$

$(x^2+y^2-x)\sqrt{x^2+y^2} = x^2 - y^2$

3 $(x^2+y^2-x)^2(x^2+y^2) = (x^2-y^2)^2$

[5] $\vec{u}\cdot\vec{v}=0$ AND $\|\vec{u}\|=\|\vec{v}\|$

$$(\vec{u}\times\vec{v})\times\vec{u} = -\vec{u}\times(\vec{u}\times\vec{v})$$ 2

$$= -[(\vec{u}\cdot\vec{v})\vec{u}-(\vec{u}\cdot\vec{u})\vec{v}]$$ 3

$$= 0\vec{u}+\|\vec{u}\|^2\vec{v}$$ 4

$$= \|\vec{v}\|^2\vec{v}$$ 2

1 $\|\vec{v}\|^2 > 0 \longrightarrow$ SAME DIRECTION AS $\vec{v}$

2 $\big\| \|\vec{v}\|^2\vec{v} \big\| = \|\vec{v}\|^2 \|\vec{v}\| = \|\vec{v}\|^3$

[6] $\vec{w} = \overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$

$= \frac{1}{2}\overrightarrow{AB} - \frac{3}{2}\overrightarrow{BC}$

$= \frac{1}{2}(\overrightarrow{AC} + \overrightarrow{CB}) - \frac{3}{2}\overrightarrow{BC}$

$= \frac{1}{2}(\vec{u} - \vec{v}) - \frac{3}{2}\vec{v} = \frac{1}{2}\vec{u} - 2\vec{v}$

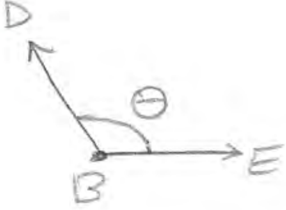
3 2 2·½ 3 ½ 1 1

[7][a] $\overrightarrow{BD} = \langle 1, 1, -4 \rangle$ 1 $\overrightarrow{BE} = \langle 0, 1, k-2 \rangle$ 1

$$\overrightarrow{BD} \cdot \overrightarrow{BE} = 0 + 1 - 4k + 8 = 9 - 4k < 0$$ 2

2

$$k > \frac{9}{4}$$ 1



[b] $\overrightarrow{AC} = \langle 1, -2, 2 \rangle$ 1

$$\text{PROJ}_{\overrightarrow{BD}} \overrightarrow{AC} = \frac{\langle 1,-2,2\rangle \cdot \langle 1,1,-4\rangle}{\langle 1,1,-4\rangle \cdot \langle 1,1,-4\rangle} \langle 1,1,-4\rangle = \frac{1-2-8}{1+1+16}\langle 1,1,-4\rangle$$

$$= \frac{-9}{18}\langle 1,1,-4\rangle$$ 2, 2

$$= \langle -\tfrac{1}{2}, -\tfrac{1}{2}, 2\rangle$$ 2

[c] $\overrightarrow{BD} \times \overrightarrow{BE} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -4 \\ 0 & 1 & k-2 \end{vmatrix} = \langle k-2+4, -(k-2), 1 \rangle$

$= \underline{\langle k+2, -k+2, 1 \rangle}$ 4½

SANITY CHECK:

$1(k+2)+1(-k+2)-4(1)$

½ $\underline{k+2-k+2-4} = 0$

$0(k+2)+1(-k+2)+(k-2)(1)$

½ $\underline{0-k+2+k-2} = 0$

$\frac{1}{2}\|\overrightarrow{BD} \times \overrightarrow{BE}\|$

$= \underline{\frac{1}{2}\sqrt{(k+2)^2+(-k+2)^2+1}}$ 2½

$= \underline{\frac{1}{2}\sqrt{2k^2+9} = \frac{5}{2}}$ 2

$2k^2+9 = 25$

$k^2 = 8 \rightarrow \underline{k = \pm 2\sqrt{2}}$ 1½

[d] $\vec{d} = \overrightarrow{AC} = \langle 1, -2, 2 \rangle$

$$x = -1 + t \qquad x = -2 + t$$
$$y = -1 - 2t \quad \text{or} \quad y = 1 - 2t$$
$$z = 1 + 2t \qquad z = -1 + 2t$$

2

[e] $\vec{d} \perp \vec{n}$ $\quad \vec{n} = \overrightarrow{BD} \times \overrightarrow{BE} = \langle k+2, -k+2, 1 \rangle$ 2

$\vec{d} \cdot \vec{n} = 0 \rightarrow 1(k+2) - 2(-k+2) + 2(1) = 0$ 2

2

$3k = 0 \rightarrow k = 0$ 1

[f] $\vec{n} = \left\langle \frac{3}{2}, \frac{5}{2}, 1 \right\rangle$ 1

$\frac{3}{2}(x-0) + \frac{5}{2}(y-0) + 1\left(z-\frac{1}{2}\right) = 0$ 3

$3x + 5y + 2z - 1 = 0$ $1\frac{1}{2}$